Mathematical Simulation (block 2.5)

Introduction

Mathematical simulation is a computer-based technique for studying properties of dynamical processes that are hard or impossible to retrieve otherwise. In particular we shall be dealing with dynamical processes involving stochastic variates (arrival processes, service times, life times, and so on).
Much attention will be given to modeling issues, both from the perspective of discrete event simulation and process simulation. Examples of problems that can be studied in this way are: queueing problems at banks or store checkouts, planning problems when loading and unloading ships and trucks, and traffic control planning.

Aim: To gain insight and experience with practical aspects of mathematical simulation.
Communication: To understand simulation projects and to be able to interpret the results.
Skills: To make conceptual models of dynamical systems and to implement and analyse simulation models based on discrete event simulation.

The course will involve the following topics:
* Introduction: types of simulation, modeling cycle, terminology.
* Discrete event simulation.
* Random numbers: generation and validation techniques.
* Statistics: hypothesis testing, confidence intervals, variance analysis.
* Output analysis: runlength analysis, variance reduction techniques.
* Metamodelling and experimental design.
* Verification and validation.
* Monte Carlo simulation.

During the course, it is possible to gain up to 1.5 bonus points, which are added to the examination result. Marks above 10 are rounded down to 10.

The first five weeks of the course involve practical sessions, involving Matlab, in the computer room 3.002.
These meetings are scheduled on Fridays. At the start of each session an assignment is handed out, with which you can earn 0.3 bonus points. For that you have to return your elaboration at the (end of the) same day, preferably to jean.derks@micc.unimaas.nl. These assignments are also found on EleUM.

The assignments are to be elaborated in Matlab. Special attention should be paid to structured coding and using short and explanatory comments.

For a nice short online guide to Matlab, see
http://www.cs.ubc.ca/spider/cavers/MatlabGuide/guide.html;
for programming style guidelines, see
http://www.datatool.com/downloads/matlab_style_guidelines.pdf;
and for writing fast MATLAB code, see
http://www.math.ucla.edu/~getreuer/matopt.pdf.

Course material: *a detailed description*

The following book is used in this course:
Averill M. Law, Simulation, Modeling and Analysis, 4th ed., McGraw-Hill International Edition, 2007.
ISBN-13: 978-0-07-110336-7
(Be aware that the third edition of this book, which features W. David Kelton as a co-author, has been significantly updated, especially Chapter 7 and many of the references to the literature.)

In this course, we have discussed selected material from the following chapters and sections.

Chapter 1 - Basic Simulation Modeling
This concerns: (i) a general introduction, (ii) the modeling cycle, (iii) discrete-event simulation, (iv) three different approaches to simulation (activity based, event based, process based), (v) queueing systems.
Sections: 1.1, 1.2, 1.3, 1.4, 1.7, 1.8, 1.9, App. 1A, App. 1B.

Chapter 4 - Review of Basic Probability and Statistics
We didn't go through this chapter systematically and in all detail, but it summarizes the background knowledge on this topic that is a prerequisite for this course. In particular it concerns details on: (i) the central limit theorem, (ii) estimation of means and variances, (iii) confidence intervals and hypothesis tests for the mean.
Sections: 4.4, 4.5.

Chapter 6 - Selecting Input Probability Distributions
This chapter contains an overview of most of the (continuous and discrete) probability density functions and distribution functions that you may encounter in the scientific literature. See Section 6.2.
It contains a description of statistical tests that you may perform to assess whether your observations are consistent with a particular given distribution. See the goodness-of-fit tests in Section 6.6.
It contains a discussion on arrival processes in general and Poisson processes in particular. Poisson processes have been discussed in some detail (exponential interarrival times, the PASTA property, ways to generate such processes).
Sections: 6.2, 6.6, 6.12.

Chapter 7 - Random-Number Generators
We have discussed important features of random-number generators, the class of LCG generators (mixed and multiplicative), cycle length issues, feedback-shift register generators (only briefly), statistical tests to test randomness issues (uniformity of the distribution, independence and correlations, runs-up test, etc.).
Sections: 7.1, 7.2, 7.3, 7.4.

Chapter 8 - Generating Random Variates
Discussed are the two most important methods for generating random variates: the inverse transform method and the acceptance-rejection method. (We have briefly exercised with the composition approach too.) Some attention was paid to sampling from the standard normal distribution (the Box-Muller method, the polar method).
Sections: 8.1, 8.2.1, 8.2.2, 8.2.4, 8.3.1, 8.3.2, 8.3.6, 8.4.7, 8.6.1.

Chapter 9 - Output Data Analysis for a Single System

Emphasis is on: (i) transient and steady-state behavior, (ii) correlation between quantities in a single run, (iii) independence across runs, (iv) estimating means, (v) estimating the warm-up length, (vi) estimating the required run length. Note that the standard deviation of an estimate is inversely proportional to the square root of the number of runs.
Sections: 9.1, 9.2, 9.3, 9.4.1, 9.5.1, 9.5.2.

Chapter 10 - Comparing Alternative System Configurations
We have discussed this topic mainly in conjunction with the "Zippy-Klunky" example (M/M/1 and M/M/2 queues). Previously discussed statistical techniques are used to set up confidence intervals.
Sections: 10.1, 10.2.1, 10.2.2.

Chapter 11 - Variance-Reduction Techniques
We have discussed this also by means of the "Zippy-Klunky" example only. Issues of importance: common random numbers, synchronization, using independent streams for different types of variables.
Sections: 11.1, 11.2.

Chapter 12 - Experimental Design and Optimization
This includes a discussion of how to build a "metamodel" of an available simulation model. Some issues: (i) the concept of a metamodel and how it relates to the given simulation model, (ii) linear regression and parameter estimation, (iii) coded variables, (iv) full factorial designs, (v) fractional factorial designs (only briefly discussed).
Sections: 12.1, 12.2, 12.3, 12.4.

We have discussed several of these issues in more depth by means of the computer assignments, which are all part of the material for this course.
(1) A simulation model of a single server queueing system, event-based, including a flowchart.
(2) Simulation of Poisson arrivals and the PASTA property (the hitchhiker's paradox).
(3) Simulation of an M/M/2 system with three types of customers to determine closing time (the barber shop).
(4) Random-number generation and testing (Midsquare method, RANDU generator, chi-square test, runs-up test).
(5) Generating random normal variates (Box-Muller and the polar method).
(6) Output variables and variance reduction: comparing M/M/1 and M/M/2 with the same utilization factor.